

# CONTROL OF THE DIRECT CURRENT ELECTRIC DRIVE BASED ON INDIRECT SPEED ESTIMATION

**S. Peresada, professor, Y. Nikonenko, Ph.D. student, V. Pyzhov, associate professor**

*Igor Sikorsky Kyiv Polytechnic Institute, department of automation of electromechanical systems and electric drives*

**Introduction.** Sensorless control is still a relevant task of the modern theory of electric drive. Many practical solutions to develop sensorless electromechanical systems with the induction and synchronous motors were recently proposed. However, the rigorous theoretical background, as well as a general methodology of the synthesis of sensorless electric drives, are absent. It is particularly relevant to the direct current (DC) electric drives.

## Review of the latest publications

Sensorless DC-drives are usually implemented on the base of thyristor DC electric drive with the feedback by electromotive force (EMF). The disadvantages of such systems are well known and difficult to be overcome when using traditional approaches to designing a control system 1. From the viewpoint of the control theory, this problem relates to the adaptive control of linear objects 2.

The general solution is to design a speed observer based on measurable signals: armature current and/or voltage. The observer in 3 uses a simplified model of DC motor, neglecting the armature inductance and current dynamics. An optimal observer 4 provides high-performance tracking only in the specific operating conditions. Sliding mode observer 5, which provides local stability of speed estimation, has a considerable level of noises in the current regulation loop. The estimation algorithm based on adaptive filters 6 implements a Kalman filter. The estimation algorithm based on torque disturbance estimation 7 is overparameterized. A hybrid fuzzy-PI observer 8 does not provide an optimal solution in all operation modes of the motor. The controller 9 provides speed estimation based on only the duration of the voltage spikes in PWM (Pulse-Width Modulation). Despite it does not require speed and current sensors, the algorithm requires an additional voltage sensor for the motor armature.

The solutions 4, 6, 9 do not provide asymptotic speed tracking. The papers 3 – 5 provide the stability analysis of the estimation convergence only, but none of them demonstrate proofs of the close-loop system stability. The solutions 4 – 9 have complex configuration procedures of the observer and controller. The configuration is strongly dependent on the DC motor parameters. The algorithms 6, 8, 9 do not provide a robustness.

The challenges, identified above can be solved based on the method proposed below, to ensure the following features to the sensorless DC drive:

- dynamic and static properties close to the system with speed measurement;
- cascaded structure of control system, which is similar to standard systems with subordinate parameters regulation;
- robustness to deviation of variables and parametric disturbances;

– simplicity of controller configuration.

**Purpose of research.** The paper aims to design a speed tracking sensorless control algorithm, which has robustness properties to coordinate deviation and parametric disturbances, based on the speed observer for DC drive with separate excitation.

**Material and results of the research.**

Formulation of the task

Assuming the exciting flux is constant, the DC motor model can be defined as

$$\begin{aligned}\dot{\omega} &= \mu i - \frac{T_L}{J}, \\ \dot{i} &= -\frac{R}{L}i + \frac{c}{L}\omega + \frac{1}{L}u,\end{aligned}\tag{1}$$

where  $\omega$  is the motor speed,  $i$  is armature current,  $u$  is control voltage,  $T_L$  is the load torque,  $R$ ,  $L$  – armature resistance and inductance,  $c$  – torque constant, and  $\mu = c/J$ .

Considering the DC motor model (1), the following assumptions are taken into consideration:

- A1. The speed reference trajectory  $\omega^*$  is smooth, bounded function together with its first  $\dot{\omega}^*$  and second  $\ddot{\omega}^*$  time derivatives;  $i^*$  is bounded current reference with bounded derivative  $\dot{i}^*$ .
- A2. The load torque  $T_L$  is unknown, constant or changing slowly and limited.
- A3. All motor parameters are known and constant.
- A4. Current  $i$  is available for measurement; speed  $\omega$  is unmeasured.

Under these assumptions, a linear dynamic controller can be found in the following form

$$\begin{aligned}u &= a(\omega^*, \dot{\omega}^*, \ddot{\omega}^*, i, z), \\ \dot{z} &= b(\omega^*, \dot{\omega}^*, \ddot{\omega}^*, i, z),\end{aligned}\tag{2}$$

which ensures asymptotic speed tracking, i.e.  $\lim_{t \rightarrow \infty} \tilde{\omega} = 0$ , where  $\tilde{\omega} = \omega - \omega^*$  – speed tracking error,  $z$  – the integral control action.

The algorithm is developed in the following stages: A) a controller with robustness properties is synthesized given speed is measured; B) the speed observer is developed; C) development of the integrated electromechanical system which consists of controller, observer, and motor. The final step D) is the stability analysis of the developed closed-loop system.

Synthesis of control algorithm

An employed output-feedback linearizing technique implies that the speed regulation loop must be designed first.

A) *Controller with the speed measurement*

From (1) the speed error dynamics becomes

$$\dot{\tilde{\omega}} = \mu \tilde{i} - \hat{\tilde{T}}_L - \tilde{T}_L - \dot{\omega}^*, \quad (3)$$

where  $\hat{\tilde{T}}_L$  is estimate of the load torque component  $T_L/J$ , and  $\tilde{T}_L = T_L/J - \hat{\tilde{T}}_L$  is the load torque estimation error.

In case of ideal current tracking and  $T_L = \text{const}$ , speed controller is described as

$$\tilde{i} = \frac{1}{\mu} \left( -k_\omega \tilde{\omega} + \hat{\tilde{T}}_L + \dot{\omega}^* \right), \quad (4)$$

$$\dot{\hat{\tilde{T}}}_L = -\dot{\tilde{T}}_L = -k_{\omega i} \tilde{\omega},$$

where  $k_\omega > 0$ ,  $k_{\omega i} > 0$  are speed controller proportional and integral gains.

The ideal dynamics of the speed loop under the current control condition is

$$\begin{aligned} \dot{\tilde{T}}_L &= k_{\omega i} \tilde{\omega}, \\ \dot{\tilde{\omega}} &= -\tilde{T}_L - k_\omega \tilde{\omega}. \end{aligned} \quad (5)$$

The system (5) is stable  $\forall (k_\omega, k_{\omega i}) > 0$ , i.e.  $\lim_{t \rightarrow \infty} (\tilde{T}_L, \tilde{\omega}) = 0$ . Its dynamic properties are set by the damping factor  $\xi$  and the natural frequency of undamped oscillations, which corresponds to the 2<sup>nd</sup> order system (5).

The armature current is not the real control action in (3), so (4) can be considered as reference  $\tilde{i}^*$  for the current  $\tilde{i}$ . Defining current tracking error as

$$\tilde{\tilde{i}} = \tilde{i} - \tilde{i}^*, \quad (6)$$

system (5) becomes

$$\begin{aligned} \dot{\tilde{T}}_L &= k_{\omega i} \tilde{\omega}, \\ \dot{\tilde{\omega}} &= -\tilde{T}_L - k_\omega \tilde{\omega} + \mu \tilde{\tilde{i}}. \end{aligned} \quad (7)$$

The control voltage must guarantee the current error tracking, i.e.  $\lim_{t \rightarrow \infty} \tilde{\tilde{i}} = 0$ . Substituting (6) into the second equation of (1), current error dynamics is

$$\dot{\tilde{\tilde{i}}} = -\frac{R}{L} \tilde{\tilde{i}} + \frac{1}{L} u - \frac{R}{L} \tilde{i}^* - \frac{c}{L} \omega - \dot{\tilde{i}}_1^* - \dot{\tilde{i}}_2^*, \quad (8)$$

where the reference current derivative is divided into the known function  $\dot{i}_1^*$  and the unknown term  $\dot{i}_2^*$  defined from (4) as

$$\begin{aligned} \dot{i}_1^* &= \frac{1}{\mu} \left( -k_\omega \left( -\tilde{T}_L - k_\omega \tilde{\omega} + \mu \tilde{i} \right) + \dot{\tilde{T}}_L + \dot{\tilde{\omega}}^* \right), \\ \dot{i}_2^* &= \frac{k_\omega}{\mu} \tilde{T}_L. \end{aligned} \quad (9)$$

The current controller is constructed from (8) and (9)

$$\begin{aligned} u &= L \left( \frac{R}{L} \dot{i}^* + \frac{c}{L} \omega + \dot{i}_1^* - k_{i1} \tilde{i} - y \right), \\ \dot{y} &= k_{i2} \tilde{i}, \end{aligned} \quad (10)$$

where  $y$  – current integral term,  $(k_{i1}, k_{i2}) > 0$  are the proportional and integral gains of the current controller.

After substitution of (10) in (8), the resulting closed-loop error dynamics is given by

$$\begin{pmatrix} \dot{\tilde{T}}_L \\ \dot{\tilde{\omega}} \\ \dot{y} \\ \dot{\tilde{i}} \end{pmatrix} = \begin{bmatrix} 0 & k_{\omega i} & 0 & 0 \\ -1 & -k_\omega & 0 & \mu \\ \hline 0 & 0 & 0 & k_{i2} \\ -k_{\omega i}/\mu & 0 & -1 & -k_{i1} \end{bmatrix} \begin{pmatrix} \tilde{T}_L \\ \tilde{\omega} \\ y \\ \tilde{i} \end{pmatrix}, \quad (11)$$

where  $k_i = R/L + k_{i1}$ .

The linear system (11) is asymptotically stable with the suitable tuning of the speed and current controllers gains  $(k_\omega, k_{\omega i}), (k_{i1}, k_{i2})$ . It is known from the theory of cascaded systems that the speed of response of the current closed-loop, given by the two last equations in (11), should be at least two times faster than speed control loop dynamics 10.

Time-scale separation between speed and current dynamics may be obtained using the standard frequency-domain approach on the base of characterization equation solution:

$$p^2 + k_p p + k_{in} = 0. \quad (12)$$

The standard adjustment of each second-order subsystem is  $k_{in} = k_p^2/4$  for  $\xi = 1$ , and  $k_{in} = k_p^2/2$  for  $\xi = 0.707$ . So that the relationship between the natural

frequency of undamped oscillations becomes  $\omega_{0i} = (2 \div 4)\omega_{0s}$ ,  $\omega_0^2 = k_{in}$ , where indexes 'i', 's' are used for current and speed loop respectively.

The resulting closed-loop error dynamics (11) has properties of structural robustness to coordinate deviation and parametric disturbances. This is due to cascaded connection of second-order systems with the two time-scale separation. The system tracks the reference speed without an error if initial conditions are at zero.

It follows from the above analysis that asymptotic speed tracking is guaranteed if assumptions A1, A2, A3 are satisfied and speed is available for measurement.

### B) Speed observer

During speed observer design, speed is assumed unmeasurable. Its output is used in the control algorithm instead of the measured speed.

Define the current and speed estimation errors as

$$e_i = i - \hat{i}, \quad e_\omega = \omega - \hat{\omega}, \quad (13)$$

and consider the following form of the speed observer:

$$\begin{aligned} \dot{\hat{\omega}} &= \mu i^* - \hat{T}_L - k_1 e_i, \\ \dot{\hat{i}} &= \frac{1}{L} \left( u - R \hat{i} - c \hat{\omega} \right) + k_2 e_i, \end{aligned} \quad (14)$$

where  $(k_1, k_2) > 0$ .

From (1) and (14), the estimation error dynamics can be described by the following equations:

$$\begin{aligned} \dot{e}_\omega &= k_1 e_i - \tilde{T}_L, \\ \dot{e}_i &= -k_{0i} e_i - \frac{c}{L} e_\omega, \end{aligned} \quad (15)$$

where  $k_{0i} = R/L + k_2$ .

Under condition  $\tilde{T}_L = 0$ , the globally stable solution of (15) is  $e_i = e_\omega = 0$ . At the same time, the observer (15) is not asymptotic due to the presence of perturbation  $\tilde{T}_L$ .

To prove the robustness properties of system (15) to  $\tilde{T}_L$ , the following coordinate transformation is considered

$$\begin{aligned} z &= k_{0i} \eta - e_\omega, \\ \eta &= \frac{L}{c} e_i. \end{aligned} \quad (16)$$

The error dynamics (15) in new coordinates (16) can be presented as

$$\begin{aligned}\dot{e}_\omega &= -\frac{k_{02}}{k_{0i}}e_\omega - \frac{k_{02}}{k_{0i}}z - \tilde{T}_L, \\ \dot{z} &= -\left(k_{0i} - \frac{k_{02}}{k_{0i}}\right)z + \frac{k_{02}}{k_{0i}}e_\omega + \tilde{T}_L,\end{aligned}\tag{17}$$

where  $k_{02} = ck_1/L$ .

Defining  $k_{02} = k_{0i}^2/2$ , equations (17) may be rewritten as

$$\begin{aligned}\dot{e}_\omega &= -\frac{k_{0i}}{2}e_\omega - \frac{k_{0i}}{2}z - \tilde{T}_L, \\ \dot{z} &= -\frac{k_{0i}}{2}z + \frac{k_{0i}}{2}e_\omega + \tilde{T}_L.\end{aligned}\tag{18}$$

From (18) it can be concluded that  $\|x(t)\| = \|x(0)\|e^{-(k_{0i}/2)t}$  under conditions of  $\tilde{T}_L = 0$ , where  $x = (e_\omega, z)^T$ . Consequently, the load disturbance can be arbitrary by increasing  $k_{0i}$ . It should be noted that the damping factor for (18) is  $\xi = 0.707$ .

### C) Integrated electromechanical system

Substituting  $\omega \rightarrow \hat{\omega}$  in (4) and (10), the estimated speed error can be defined as

$$\tilde{\omega} = \hat{\omega} - \omega^*.\tag{19}$$

The speed controller (14) can be represented in the following form:

$$\begin{aligned}\dot{i}^* &= \frac{1}{\mu}\left(-k_\omega\tilde{\omega} + \hat{T}_L + \dot{\omega}^*\right), \\ \dot{\hat{T}}_L &= -\tilde{T}_L = -k_{\omega i}\tilde{\omega}.\end{aligned}\tag{20}$$

Taking into account that  $\tilde{\omega} = \hat{\omega} - e_\omega$ , the system (7) can be rewritten as

$$\begin{aligned}\dot{\hat{T}}_L &= k_{\omega i}\tilde{\omega} - k_{\omega i}e_\omega, \\ \dot{\hat{\omega}} &= -\tilde{T}_L - k_\omega\tilde{\omega} + \mu\dot{i}^* + k_\omega e_\omega.\end{aligned}\tag{21}$$

The current controller is formed similarly to (10) in the following form:

$$\begin{aligned}u &= L\left(\frac{R}{L}\dot{i}^* + \frac{c}{L}\omega^* + \dot{i}^* - k_{ii}\tilde{i} - y\right), \\ \dot{y} &= k_{ii}\tilde{i}.\end{aligned}\tag{22}$$

The current derivative  $\dot{i}^*$  is known from the solution of the equation (20)

$$\begin{aligned} \dot{i}^* &= \frac{1}{\mu} \left( -k_\omega \dot{\tilde{\omega}} + \dot{\hat{T}}_L + \ddot{\omega}^* \right), \\ \dot{\tilde{\omega}} &= -k_\omega \tilde{\omega} - k_1 e_1. \end{aligned} \quad (23)$$

Substituting the current control algorithm (22), (23) to (8), and taking into account (18) and (21), the resulting tracking and estimation error dynamics becomes

$$\begin{pmatrix} \dot{\tilde{T}}_L \\ \dot{\tilde{\omega}} \\ \dot{y} \\ \dot{\tilde{i}} \\ \dot{e}_\omega \\ \dot{z} \end{pmatrix} = \begin{bmatrix} 0 & k_{\omega i} & 0 & 0 & -k_{\omega i} & 0 \\ -1 & -k_\omega & 0 & 0 & k_\omega & 0 \\ \hline 0 & 0 & 0 & k_{ii} & 0 & 0 \\ 0 & 0 & -1 & -k_i & -c/L & 0 \\ \hline -1 & 0 & 0 & 0 & -k_{0i}/2 & -k_{0i}/2 \\ 1 & 0 & 0 & 0 & k_{0i}/2 & -k_{0i}/2 \end{bmatrix} \begin{pmatrix} \tilde{T}_L \\ \tilde{\omega} \\ y \\ \tilde{i} \\ e_\omega \\ z \end{pmatrix}. \quad (24)$$

The equation system (24) represents a cascaded connection of three subsystems: speed regulation loop, current regulation loop, and speed observer. If the observer response is at least 3–4 times faster than the current loop, dynamics of the proposed system is similar to the system with the speed measurement. This corresponds to the following natural frequency of undamped oscillations:  $\omega_{0o} = (3 \div 4) \omega_{0i}$ ,  $\omega_{0o}^2 = k_{0i}^2/2$ , where index ‘o’ is used for observer dynamics. Note that it was assumed  $c/L \ll k_i$ , that is usually taking place in practice.

The block-diagram of developed speed tracking algorithm is demonstrated in Figure 1. It consists of the PI-speed controller (20), PI-current controller (22), and the speed observer (14).

#### D) Simulation results

The proposed control algorithm was applied for DC motor, whose rated data are  $P_N = 500$  W,  $R = 1$  Ohm,  $L = 5$  mH,  $J = 0.01$  kg m<sup>2</sup>,  $c = 1$  Nm/A.

The controllers parameters were set at:  $k_\omega = 200$ ,  $k_{\omega i} = k_\omega^2/2$ ,  $k_i = 1000$ ,  $k_{ii} = k_i^2/2$ ; tuning parameters of the speed observer (14) are  $k_2 = 2000$ ,  $k_1 = k_2^2/2$ .

The operating sequence is following: at the initial time the unloaded motor is required to track the speed reference trajectory  $\omega^*(t)$ , starting from zero initial value and reaching rated value at time  $t = 0.15$  s. The required dynamic torque equals to double rated value. At time  $t = 0.3$  s, a constant load torque, equal to the motor rated value, is applied; at time  $t = 0.5$  s load torque is set to zero. Transients of speed trajectory tracking are depicted in Figure 2. It goes from Figure 2 that the speed tracking error only occurs at the moments of applying the load. Transient  $\tilde{\omega}(t)$  is virtually the same as for the algorithm (4), (9) with speed sensor.

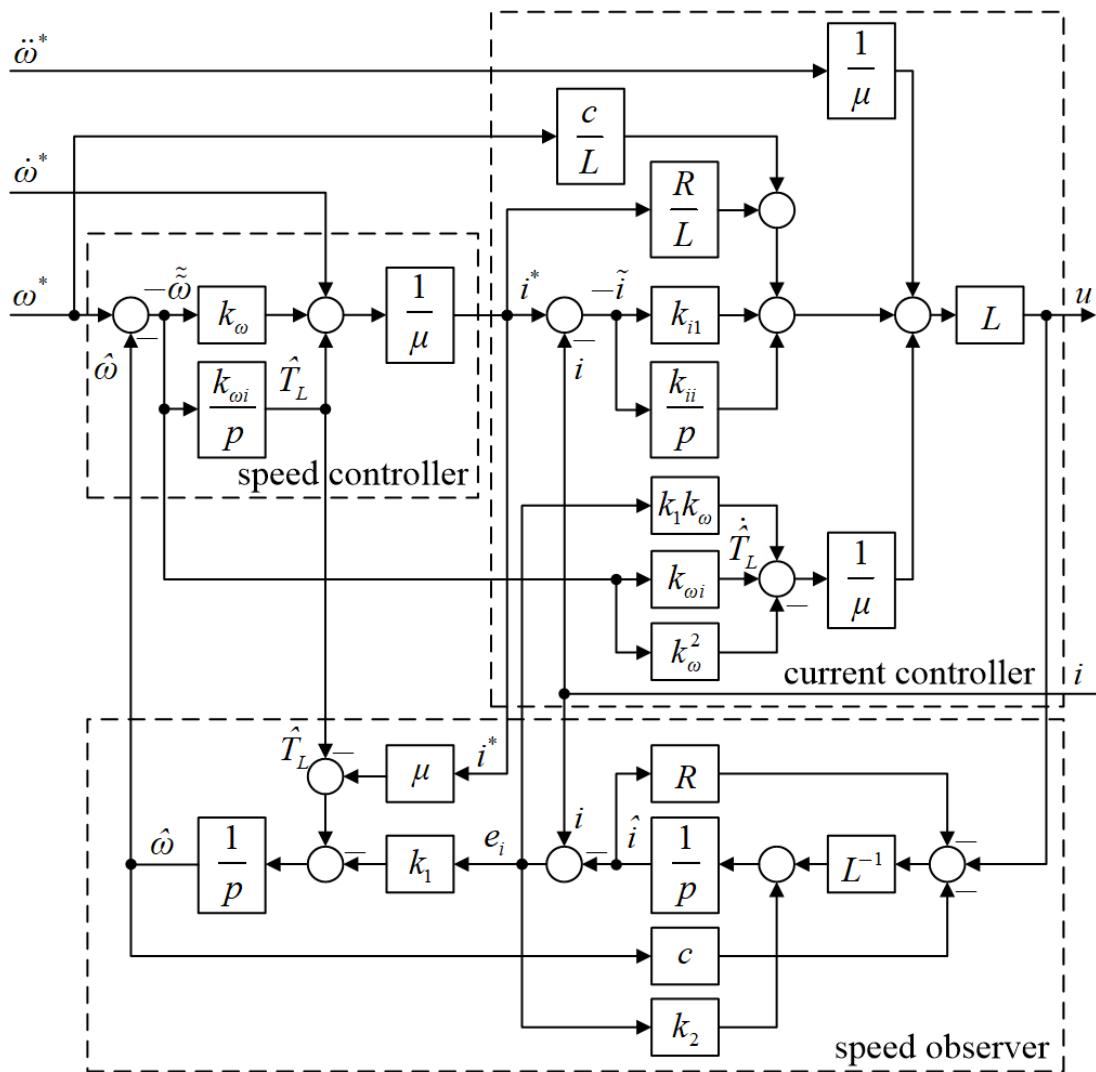


Figure 1 – Block-diagram of control algorithm

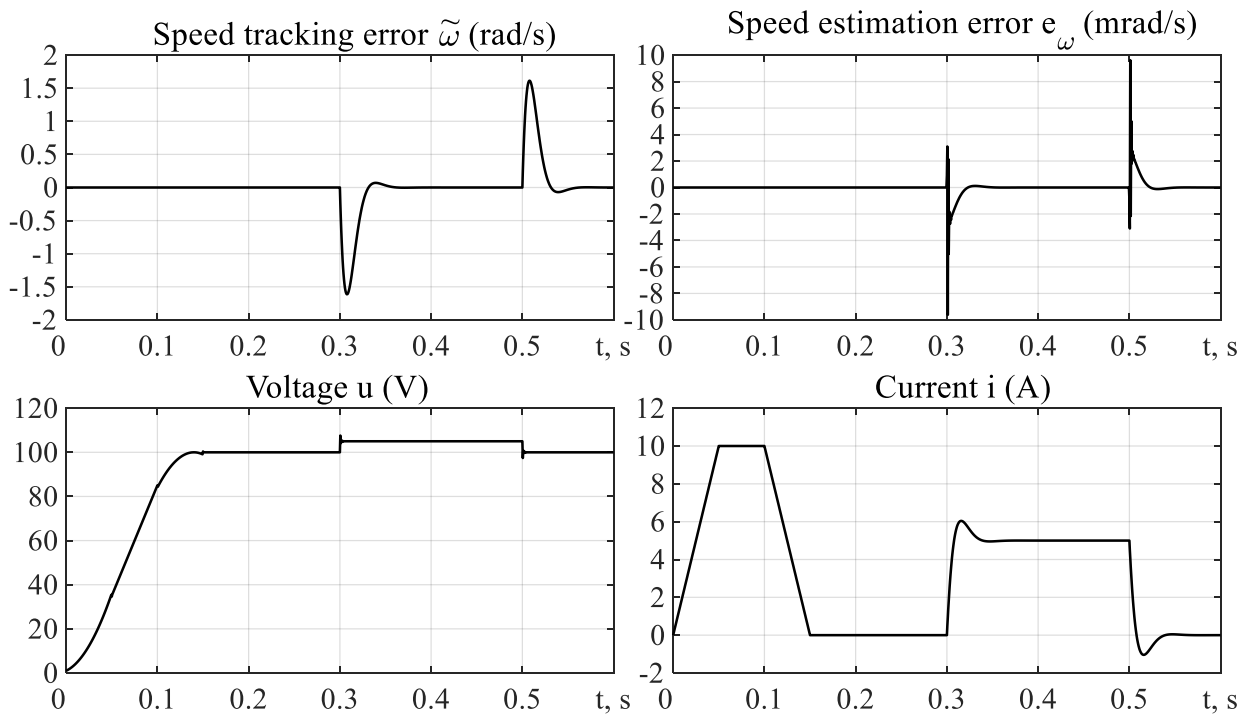


Figure 2 – Transients of speed trajectory tracking



**Conclusions.** A control algorithm for speed trajectory tracking is designed for DC drive, which not requires the direct speed measurement. The algorithm provides dynamic and static properties close to the system with a speed sensor and ensures system robustness to parametric disturbances and deviations of variables.

### References

1. Rajashekara K., Kawamura, A., Matsuse, K. (1996). Sensorless control of AC motor drives: speed and position sensorless operation. New York: IEEE press.
2. Potapenko, Y., et al. (1998). Robust control system of electric drive. Bulletin of National Technical University "Kharkiv Polytechnic Institute": Problems of Automated Electric Drives. Theory and Practice. Power Electronics and Energy Efficiency, 60, pp. 121 – 122. (in Russian)
3. Liu, Z. Z., Luo, F. L., Rashid, M. H. (2003). Speed nonlinear control of DC motor drive with field weakening. *IEEE Transactions on Industry Applications*, 39(2), 417-423.
4. Leephakpreeda, T. (2002). Sensorless DC motor drive via optimal observer-based servo control. *Optimal Control Applications and Methods*, 23(5), 289-301.
5. Ahmadi, A. A., Salmasi, F. R., Noori-Manzar, M., Najafabadi, T. A. (2013). Speed sensorless and sensor-fault tolerant optimal PI regulator for networked DC motor system with unknown time-delay and packet dropout. *IEEE Transactions on Industrial Electronics*, 61(2), 708-717.
6. Hilairret, M., Auger, F. (2007). Speed sensorless control of a DC-motor via adaptive filters. *IET Electric Power Applications*, 1(4), 601-610.
7. Buja, G. S., Menis, R., Valla, M. I. (1995). Disturbance torque estimation in a sensorless DC drive. *IEEE transactions on industrial electronics*, 42(4), 351-357.
8. Gunde, A., Devadasu, G., Vijayasaanthi, M. (2019). Design of hybrid fuzzy-PI Controller for sensorless speed control of separately excited DC motor drive. *International Journal of Recent Technology and Engineering (IJRTE)*, 8(258), 1189-1192.
9. Radcliffe, P., Kumar, D. (2015). Sensorless speed measurement for brushed DC motors. *IET Power Electronics*, 8(11), 2223-2228.
10. Narendra, K. S., Annaswamy, A. M. (1989). Stable adaptive systems. Upper Saddle River, NJ: Prentice Hall.