РОЗДІЛ 5. АВТОМАТИЗАЦІЯ ЕЛЕКТРОМЕХАНІЧНИХ СИСТЕМ ТА ЕЛЕКТРОПРИВОД

SURVEY OF THE INTERIOR PERMANENT MAGNET SYNCHRONOUS MOTOR MODELS CONSIDERING SATURATION AND CROSS-MAGNETIZATION

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Introduction. Interior permanent magnet synchronous motors (IPMSMs) industrial implementation is recently increased due to its rigid construction as well as high efficiency and high power factor. Especially IPMSMs are used in electric vehicles (EVs) due to their high power and torque density [1].

High precision and high dynamic applications require exact knowledge of motor parameters during operation. The parameters of the model vary nonlinearly depending on operation conditions, which change level of saturation and magnetic field distribution in the machine [2]. Iron saturation changes the air gap flux density, causing inductances of the machine to vary.

Saturation and hence inductance variation and cross-coupling effects are considerable in the IPMSM due to its geometry, therefore it cannot be neglected when creating highly efficient control algorithms. Control algorithm synthesis is based on motor model, thus, its proper selection is vital especially if system is highly nonlinear. Problem of model derivation for the IPMSM considering saturation and cross-coupling remains not fully solved as it is unique for each motor depending on its constructive features.

Most of the researches start model derivation from flux linkage model [1] – [9]. In [1] inductances are determined using finite element analysis (FEA), influence of mutual inductances on electromagnetic torque and steady state model is considered. Similar approach is used in [8] but cross-coupling is presented as additional flux components. In [5] lookup tables are used to determine currents from flux linkages model. In [2] author proposes to use flux linkage model directly, determining dynamic inductances from flux values by means of differentiation. Model in [6] is similar to proposed in [2] but flux is determined using observer. In [7] FEA is used, but flux from permanent magnets is a function of both currents. The other approach is to use linearized model considering that inductances are not constants, which is not mathematically correct.

Purpose. To derive IPMSM models considering saturation and cross-coupling effects with different degree of simplification. To compare the models with the existed ones.

Material and results of the research. General model that consider saturation and cross-coupling effects in the IPMSM can be expressed in flux linkage terms as currents functions [1] – [9]. Equations of the model in d-q axis rotor reference frame are presented below.

$$
\begin{aligned}
u_d &= Ri_d + \frac{d\psi_d\left(i_d, i_q\right)}{dt} - p_n\omega\psi_q\left(i_d, i_q\right),\\
u_q &= Ri_q + \frac{d\psi_q\left(i_d, i_q\right)}{dt} + p_n\omega\psi_d\left(i_d, i_q\right),\\
\frac{d\omega}{dt} &= \frac{3p_n}{2J}\left(\psi_d\left(i_d, i_q\right)i_q - \psi_q\left(i_d, i_q\right)i_d\right) - \frac{F}{J}\omega - \frac{T_L}{J},
\end{aligned}
\tag{1}
$$

where R – stator resistance, $\left(i_d, i_q\right)$ - direct and quadrature axis currents respectively, $\left(\psi_d\left(i_d, i_q\right), \psi_q\left(i_d, i_q\right)\right)$ - direct and quadrature axis flux linkages respectively, $\left(u_d, u_q\right)$ - direct and quadrature axis voltages respectively, ω - mechanical speed, p_n - pole pairs number, J - moment of inertia, F – viscous friction, T_L - load torque.

Assuming that self and cross inductances are considered as functions of both currents (i_d and i_q), flux equations are

$$
\begin{bmatrix} \psi_d\left(i_d, i_q\right) \\ \psi_q\left(i_d, i_q\right) \end{bmatrix} = \begin{bmatrix} L_d\left(i_d, i_q\right) & L_{dq}\left(i_d, i_q\right) \\ L_{qd}\left(i_d, i_q\right) & L_q\left(i_d, i_q\right) \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \psi_M \\ 0 \end{bmatrix},
\tag{2}
$$

where $L_d\left(i_d, i_q\right)$ - d - axis self-inductance, $L_q\left(i_d, i_q\right)$ - q - axis self-inductance, $L_{dq}\left(i_d, i_q\right)$ - d - axis mutual inductance, $L_{qd}\left(i_d, i_q\right)$ - q - axis mutual inductance, ψ_M - flux linkage due to permanent magnets.

Model (1) allows to describe saturation and cross-coupling effects in the IPMSM, but it cannot be used for closed loop control synthesis due to complexity of flux measurement. In most of the cases model (1) has to be transformed to a model with explicit current derivatives [2], [6], [7].

Models derivations

Model of the IPMSM if self-inductances and mutual inductances are functions of both currents. In this case, direct axis flux linkage derivative has to be expressed as derivative of complex function.

$$
\frac{d\psi_d\left(i_d, i_q\right)}{dt} = \frac{di_d}{dt}\frac{\partial\psi_d\left(i_d, i_q\right)}{\partial i_d} + \frac{di_q}{dt}\frac{\partial\psi_d\left(i_d, i_q\right)}{\partial i_q}.
\tag{3}
$$

Analogically to (3), quadrature axis flux linkage derivative is

$$\frac{d\psi_q(i_d,i_q)}{dt}=\frac{di_q}{dt}\frac{\partial\psi_q(i_d,i_q)}{\partial i_q}+\frac{di_d}{dt}\frac{\partial\psi_q(i_d,i_q)}{\partial i_d}. \tag{4}$$

Substitution of (3) and (4) into (1) gives

$$\begin{aligned}
&\frac{d\omega}{dt}=\frac{3p_n}{2J}\left(\psi_d(i_d,i_q)i_q-\psi_q(i_d,i_q)i_d\right)-\frac{F}{J}\omega-\frac{T_L}{J},\\
&\frac{di_d}{dt}\frac{\partial\psi_d(i_d,i_q)}{\partial i_d}+\frac{di_q}{dt}\frac{\partial\psi_d(i_d,i_q)}{\partial i_q}=-Ri_d+\psi_q(i_d,i_q)p_n\omega+u_d,\\
&\frac{di_q}{dt}\frac{\partial\psi_d(i_d,i_q)}{\partial i_d}+\frac{di_d}{dt}\frac{\partial\psi_d(i_d,i_q)}{\partial i_q}=-Ri_q-\psi_d(i_d,i_q)p_n\omega+u_q.
\end{aligned} \tag{5}$$

Obtained model (5) is the same as presented in [2], [6], [7]. Model allows to describe all processes concerning saturation and cross coupling if flux linkages $\psi_q(i_d,i_q)$ and $\psi_d(i_d,i_q)$ are known. If $\psi_M=\text{const}$, partial derivatives of flux linkages by currents from are

$$\begin{aligned}
&\frac{\partial\psi_d(i_d,i_q)}{\partial i_d}=L_d(i_d,i_q)+i_d\frac{\partial L_d(i_d,i_q)}{\partial i_d}+i_q\frac{\partial L_{dq}(i_d,i_q)}{\partial i_d},\\
&\frac{\partial\psi_d(i_d,i_q)}{\partial i_q}=L_{dq}(i_d,i_q)+i_d\frac{\partial L_d(i_d,i_q)}{\partial i_q}+i_q\frac{\partial L_{dq}(i_d,i_q)}{\partial i_q},\\
&\frac{\partial\psi_q(i_d,i_q)}{\partial i_q}=L_q(i_d,i_q)+i_q\frac{\partial L_q(i_d,i_q)}{\partial i_q}+i_d\frac{\partial L_{qd}(i_d,i_q)}{\partial i_q},\\
&\frac{\partial\psi_q(i_d,i_q)}{\partial i_d}=L_{qd}(i_d,i_q)+i_q\frac{\partial L_q(i_d,i_q)}{\partial i_d}+i_d\frac{\partial L_{qd}(i_d,i_q)}{\partial i_d}.
\end{aligned} \tag{6}$$

In the equation (6) flux derivatives are split into three components, however information about each component does not lead to advantages from control synthesis point of view. The derivatives can be considered as dynamic inductances, which fully describe magnetizing processes during transients as proposed in [2].

Substitution (6) and (2) into (5) allows to obtain model that is free from flux linkages. Drawbacks are complicity, as well as necessity to have data about self and mutual inductances and its derivatives. Model (5) is more preferable to use from that point of view as it allows to describe all processes concerning saturation and cross coupling requiring less amount of data.

Model of the IPMSM if self-inductances are functions of both currents, cross-coupling is neglected. Cross saturation can be neglected unless currents are extremely high [4], [6]. In this case mutual inductances $L_{dq}\left(i_d, i_q\right)$ and $L_{qd}\left(i_d, i_q\right)$ are taken as zero in. Then model from (5) is transformed to

$$
\begin{aligned}
&\frac{d\omega}{dt} = \frac{3p_n}{2J}\left(\left(L_d\left(i_d, i_q\right) - L_q\left(i_d, i_q\right)\right)i_d i_q + \psi_m i_q\right) - \frac{F}{J}\omega - \frac{T_L}{J}, \\
&\frac{di_d}{dt}\left(L_d\left(i_d, i_q\right) + i_d \frac{\partial L_d\left(i_d, i_q\right)}{\partial i_d}\right) + \frac{di_q}{dt}\left(i_d \frac{\partial L_d\left(i_d, i_q\right)}{\partial i_q}\right) = \\
&= -Ri_d + p_n \omega L_q\left(i_d, i_q\right) i_q + u_d, \qquad (7) \\
&\frac{di_q}{dt}\left(L_q\left(i_d, i_q\right) + i_q \frac{\partial L_q\left(i_d, i_q\right)}{\partial i_q}\right) + \frac{di_d}{dt}\left(i_q \frac{\partial L_q\left(i_d, i_q\right)}{\partial i_d}\right) = \\
&= -Ri_q - p_n \omega\left(L_d\left(i_d, i_q\right) i_d + \psi_M\right) + u_q.
\end{aligned}
$$

In model (7) d – and q – axis contours are still coupled with derivatives. In [6] derivatives of inductances by opposite currents are neglected. Also, dynamic inductances are considered as function of both currents, self-inductances – functions of one current. In this case d – and q – axis contours are decoupled and model becomes significantly simpler.

Model of the IPMSM if self-inductances are functions of one correspond current and cross-coupling is neglected. In order to simplify (7), self-inductances can be considered as functions of their own current. Model (7) in this case can be rewritten as

$$
\begin{aligned}
&\frac{d\omega}{dt} = \frac{3p_n}{2J}\left(\left(L_d\left(i_d\right) - L_q\left(i_q\right)\right)i_d i_q + \psi_m i_q\right) - \frac{F}{J}\omega - \frac{T_L}{J}, \\
&\frac{di_d}{dt}\left(L_d\left(i_d\right) + i_d \frac{\partial L_d\left(i_d\right)}{\partial i_d}\right) = -Ri_d + p_n \omega L_q\left(i_q\right) i_q + u_d, \qquad (8) \\
&\frac{di_q}{dt}\left(L_q\left(i_q\right) + i_q \frac{\partial L_q\left(i_q\right)}{\partial i_q}\right) = -Ri_q - p_n \omega\left(L_d\left(i_d\right) i_d + \psi_M\right) + u_q.
\end{aligned}
$$

Model (8) is comparably simple, and saturation is partially considered. Attempts to simplify model (8) lead to obtaining conventional IPMSM model when inductances are constants and cross-coupling is neglected:

$$\begin{aligned}
&\frac{d\omega}{dt}=\frac{3p_n}{2J}\left(\left(L_d-L_q\right)i_d i_q+\psi_M i_q\right)-\frac{F}{J}\omega-\frac{T_L}{J},\\
&L_d\frac{di_d}{dt}=-Ri_d+p_n\omega L_q i_q+u_d,\\
&L_q\frac{di_q}{dt}=-Ri_q-p_n\omega\left(L_d i_d+\psi_M\right)+u_q.
\end{aligned}\tag{9}$$

It is the simplest model that is used for control algorithm synthesis, but it poorly describes motor behaviour.

Conclusion. Saturation and cross-coupling effects in IPMSMs have huge impact in motor behaviour, therefore, cannot be neglected in control algorithm synthesis. In this paper IPMSM models considering saturation and cross-coupling with different degree of simplification were derived and compared to existing ones. Selection of model to describe motor behaviour depends on motor features and essentially is compromise between model simplicity and accuracy.

References

1. Li Huimin, A Novel Optimal Current Trajectory Control Strategy of IPMSM Considering the Cross Saturation Effects / Huimin Li, Jian Gao, Shoudao Huang, Peng Fan // Energies. – 2017. – №10. – P. 1460-1476.

2. Stumberger B., Evaluation of saturation and cross-magnetization effects in interior permanent-magnet synchronous motor / B. Stumberger, G. Stumberger, D. Dolinar, A. Hamler and M. Trlep // IEEE Transactions on Industry Applications. – 2003. – Vol. 39. – №5. – P. 1264-1271.

3. Consoli A., Steady-State and Transient Operation of IPMSMs Under Maximum-Torque-per-Ampere Control / A. Consoli, G. Scarcella, G. Scelba and A. Testa // IEEE Transactions on Industry Applications. – 2010. – Vol. 46. – №1. – P. 121-129.

4. Kim Hyunbae, A novel method for initial rotor position estimation for IPM synchronous machine drives / Hyunbae Kim, Kum-Kang Huh, R. D. Lorenz and T. M. Jahns // IEEE Transactions on Industry Applications. – 2004. – Vol. 40. – №5. – P. 1369-1378.

5. Matzen T. N. Modelling magnetic saturation effects in IPMSMs for use in sensorless saliency based methods / T. N. Matzen and P. O. Rasmussen // European Conference on Power Electronics and Applications – 2007 – P. 1-8

6. Jeong Y. Adaptive Flux Observer with On-line Inductance Estimation of an IPMSM Considering Magnetic Saturation / Y. Jeong and S. Sul // IEEE 36th Power Electronics Specialists Conference – 2005 – P. 2467-2473

7. Li Z. MTPA control of PMSM system considering saturation and cross-coupling / Z. Li and H. Li // 15th International Conference on Electrical Machines and Systems (ICEMS). – 2012 – P. 1-5

8. Chedot L. Integrated Starter Generator: The Need for an Optimal Design and Control Approach. Application to a Permanent Magnet Machine / L. Chedot, G. Friedrich, J. Biedinger and P. Macret // IEEE Transactions on Industry Applications. – 2007 – vol. 43. – № 2 – P. 551-559.

9. Pellegrino G. The Rediscovery of Synchronous Reluctance and Ferrite Permanent Magnet Motors / Pellegrino, Gianmario & Jahns, Thomas & Bianchi, Nicola & Soong, Wen & Cupertino, Francesco. – 2016.